A Novel Optimal Trajectory Planning for Overhead Traveling Crane

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Abstract

The efficiency of overhead traveling crane transportation is limited by payload swing induced by velocity change of the crane. To obtain high efficiency, this study introduced an ideal trajectory for crane's speed trajectory tracking controller. Pontryagin's maximum principle was used to develop an optimal trajectory to minimize the swing angle of payload based on the kinematic model of overhead traveling crane with constant hoist rope length. A simplified trajectory which is more convenience for engineering practice is introduced and some properties of the optimal trajectory are discussed by using the simplified version. Theoretical analysis has shown that the proposed optimal trajectory and its simplified version can reach a given velocity in a given time with a minimum payload swing. Methods introduced can be applied to motion planning for overhead traveling crane.

Keywords

Optimal Control; Ideal Trajectory; Anti-swing Control; Overhead Traveling Crane

Introduction

For its large capacity and high transport efficiency, overhead traveling cranes have been widely used in industry to transfer heavy objects. However, crane systems have intrinsic drawbacks: since loads are moved via flexible ropes, horizontal motion would induce undesirable load swing. The trolley requires accelerating to a certain velocity within a short time to increase transportation efficiency. At the same time, higher acceleration would increase payload swing, which would lower transport accuracy, efficiency and cause safety hazard. So, payload swing should be suppressed as much as possible to increase transportation efficiency.

Extensive research has been made to develop control techniques toward anti-sway load transfer. As an overhead crane is an under actuated system, the payload swing could not be controlled directly. In practice, load swing is usually suppressed by trolley motion. Many control laws have been proposed to stabilize and minimize the payload swing such as simple feedback controls, input shaping techniques, energy-based controllers and fuzzy logic controllers. However, only a little attention has been paid to motion planning, that is a kinematic problem, finding certain anti-swing trolley motions for given payload motions without considering required forces that cause such motions. In studying motion planning of the crane system, the desired trajectory of the trolley is very important in achieving satisfactory performance. Ideal trajectory can be designed which would induce minor payload swing when trolley is accelerated to certain speed. Then control techniques can be adopted in the overhead crane system to track the ideal trajectory.

A new motion planning method is proposed in this study which in ideal conditions, could tame the payload swing into a controllable manner in acceleration zone and suppress swing angle to zero at the beginning of constant-velocity zone. Some properties an ideal trajectory requires are discussed. Then a motion-planning method which would generate de-sired trajectory include those above properties is introduced and analysed.

Overhead Crane Model Page Layout

The planar model of an overhead traveling crane system is shown in Fig. 1.

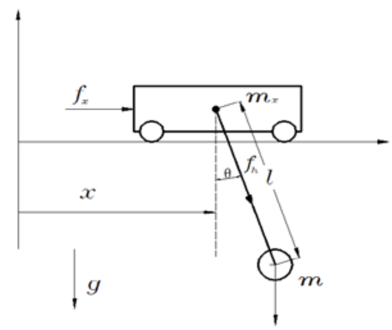

FIG. 1 OVERHEAD CRANE PLANAR MODEL

where x, l, and θ are the trolley position, hoisting rope

length and swing angle, respectively. Several assumptions are made in this model: the hoisting rope is massless and rigid link the payload is considered as a point mass; the swing angle of the payload remains in the interval between -π/2 and π/2.

The dynamics of this model is described in the following equations:

$$f_h = m\dot{\theta}^2 l + mg\cos\theta - mg\sin\theta \tag{1}$$

$$m_x\ddot{x} = f_x + f_h\sin\theta - f_{dx} \tag{2}$$

$$m\ddot{x}\cos\theta + mg\sin\theta + m\ddot{\theta}l = 0 \tag{3}$$

Where m —payload mass; m_x —trolley mass; f_x —driving force exerted on the trolley; f_h —extensile force on the hoisting rope; f_{dx} —disturbances on trolley in x direction; g—gravitational acceleration.

As stated in introduction, this paper focuses on motion planning in the overhead traveling crane system. In the mathematical model above, equation (3) alone could describe the relationship between trolley motion and payload swing angle for a given hoist rope length l. Equation (3) can be rewritten as following:

$$\ddot{\theta}l + \ddot{x}\cos\theta + \mathrm{g}\sin\theta = 0 \tag{4}$$

When the swing angle is small enough, the equation (4) can be linearized as following:

$$\ddot{\theta}l + \ddot{x} + \mathrm{g}\theta = 0 \tag{5}$$

In the kinematic model above, the swing angle is considered as the output. And it is noted that the only input is the trolley acceleration and the output is independent of load mass.

Motion Planning Objective

In practice, overhead traveling crane motion can be divided into five phases: up lifting phase, acceleration phase, constant velocity phase, deceleration phase and unloading phase. Horizontal displacement occurs in acceleration phase, constant velocity phase and deceleration phase. To attain high efficiency, acceleration and deceleration time is required to be as short as possible within certain constraints. For convenience, this study rewrites the object into its equivalent statement: to reach certain velocity within a given time and cause minimum swing. In reaching a given velocity within a given time, a desired trajectory would induce a minimum swing in acceleration phase and damp the swing into zero when entering the constant velocity phase. In most overhead traveling cranes, the trolley can move at various speeds in a given range, and many of them are determined manually by the operator. To obtain different constant motion velocity using the same acceleration motion template, the acceleration contains a uniform acceleration segments which could remain constant acceleration for an arbitrary time. In the deceleration phase, the start and end state are already known. So an optimal trajectory can be obtained with maximum principle

Proposed Motion Planning

In this section, acceleration phase and deceleration phase will be mainly discussed. For the reason described in last section, trajectory in those two phases would be different.

At the beginning of the transportation, the trolley will accelerate to a constant velocity in the acceleration phase. In the proposed motion scheme, acceleration phase is further divided into three segments: jerk period, uniform acceleration period and swing damping period.

In jerk period, swing angle would increase from 0 to θ_a, while at the end of jerk period and revert to zero. An optimal trajectory for this period will be obtained using the Pendragon's principle of maximum. The state variables, control variable and their relation are introduced as below:

$$x_1 = \theta, x_2 = \dot{\theta}, x_3 = \ddot{\theta}, u = \dddot{\theta} \tag{6}$$

$$\dot{x}_1 = x_2, \dot{x}_2 = x3, \dot{x}_3 = u \tag{7}$$

To obtain the optimal acceleration in jerk period trajectory, a unique control is conditioned by the requirement:

$$J = \int_0^{t_s} \frac{1}{2}(x_1^2 + u^2)dt \to \min \tag{8}$$

This condition of optimality prevents the change of swing angular acceleration and swing angle from becoming too high, then Hamiltonian function can be obtained:

$$H = \frac{1}{2}(x_1^2 + u^2) + \lambda_1 x_2 + \lambda_2 x_3 + \lambda_3 u \tag{9}$$

Where variables $\lambda_1, \lambda_2, \lambda_3$ satisfy the differential equations:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1}; \dot{\lambda}_2 = -\frac{\partial H}{\partial x_2}; \dot{\lambda}_3 = -\frac{\partial H}{\partial x_3} \tag{10}$$

According to the required condition of extreme in the theorem of the principle of maximum $\partial H/\partial u = 0$, equation below is obtained:

$$u = -\lambda_3 \tag{11}$$

By combining equation (7), (10) and (11), the following equation system is obtained:

$$\dot{x}_1 = x_2; \dot{x}_2 = x3; \dot{x}_3 = u;$$
$$\dot{\lambda}_1 = -x_1; \dot{\lambda}_2 = -\lambda_1; \dot{\lambda}_3 = -\lambda_2; \tag{12}$$

Solving the equation system (12), has the following forms:

$$x_1(t) = (C_3 e^{-\frac{1}{2}t} + C_5 e^{\frac{1}{2}t})\sin(\frac{\sqrt{3}}{2}t)$$
$$+ (C_4 e^{-\frac{1}{2}t} + C_6 e^{\frac{1}{2}t})\cos(\frac{\sqrt{3}}{2}t) + C_1 e^{t} + C_2 e^{-t} \tag{13}$$

$$x_2(t) = \frac{1}{2}\left[(C_3\sqrt{3} - C_4)e^{-\frac{1}{2}t} + (C_5\sqrt{3} + C_6)e^{\frac{1}{2}t}\right]\cos(\frac{\sqrt{3}}{2}t)$$
$$\frac{1}{2}\left[(-C_3 - C_4\sqrt{3})e^{-\frac{1}{2}t} + (C_5 - C_6\sqrt{3})e^{\frac{1}{2}t}\right]\sin(\frac{\sqrt{3}}{2}t) + C_1 e^{t} - C_2 e^{-t} \tag{14}$$

$$x_3(t) = \frac{1}{2}\left[(-C_3\sqrt{3} - C_4)e^{-\frac{1}{2}t} + (C_5\sqrt{3} + C_6)\right]\cos(\frac{\sqrt{3}}{2}t)$$
$$+\frac{1}{2}\left[(-C_3 + C_4\sqrt{3})e^{-\frac{1}{2}t} - (C_5 + C_6\sqrt{3})e^{\frac{1}{2}t}\right]\sin(\frac{\sqrt{3}}{2}t) + C_1 e^{t} + C_2 e^{-t} \tag{15}$$

$$u(t) = (C_3 e^{-\frac{1}{2}t} - C_5 e^{\frac{1}{2}t})\sin(\frac{\sqrt{3}}{2}t)$$
$$+ (C_4 e^{-\frac{1}{2}t} - C_6 e^{\frac{1}{2}t})\cos(\frac{\sqrt{3}}{2}t) + C_1 e^{t} - C_2 e^{-t} \tag{16}$$

Expressions for integration constants $C_i (i = 1,...,6)$ are defined by boundary conditions below:

$$\theta(0) = 0\ ; \dot{\theta}(0) = 0\ ;\ \ddot{\theta}(0) = 0$$
$$\theta(t_j) = \theta_a\ ; \dot{\theta}(0) = 0\ ;\ \ddot{\theta}(0) = 0$$

Because of their complexity, those constants are not given in the analytical form.

In uniform acceleration period, swing angle and trolley acceleration remain constant θ_a and a_a. When the trolley's velocity is near desired value, it will enter the swing damping period which is a reverse motion of the jerk period. The time span of swing damping period is the same with jerk period. At the end of swing damping period, the swing angle and both first and second order derivative of the swing angle will return to zero, and the trolley enter the constant velocity phase for an arbitrary time t_c until the object is near its destination.

Finally, the trolley will come to a stop in the deceleration phase. In this phase, the motion will be controlled as such that it will decrease the given constant velocity v_c into zero and cause a minimal load swing and damp the swing angle back to zero when trolley stop. As the start and end state of the system are determined in the deceleration phase, the trajectory can be obtained using optimal method.

The initial state is $\theta(0) = 0, \dot{\theta}(0) = 0, \dot{x}(0) = v_c$ and desired end state is $\theta(t_s) = 0, \dot{\theta}(t_s) = 0, \dot{x}(t_s) = 0$.

In "Optimal Control of Motion of the System Based on Mathematical Pendulum with Constant Length", a complicate result is obtained with a unique control. To avoid the complication and the linearization involved in the derivation of ideal trajectory using optimal method, equation (5) is transformed as follow:

$$\dot{x}(t_s) - \dot{x}(0) = -\dot{\theta}(t_s)l - \int_0^{t_s} g\theta(t)dt$$

As $\dot{\theta}(t_s) = 0$ and $\dot{x}(t_s) = 0$, the following statement is obtained:

$$\dot{x}(0) = \int_0^{t_s} g\theta(t)dt \tag{17}$$

The state variables, control variable and their relation are introduced as below:

$$\int_0^t \theta dt = x_1; \theta = x_2; \dot{\theta} = x_3; \ddot{\theta} = u; \tag{18}$$

$$\dot{x}_1 = x_2; \dot{x}_2 = x3; \dot{x}_3 = u \tag{19}$$

The boundary condition is as following:

$$x_1(0) = 0; x_2(0) = 0; x_3(0) = 0; \tag{20a}$$

$$x_1(t_s) = \frac{v_c}{g}; x_2(t_s) = 0; x_3(t_s) = 0; \tag{20b}$$

The unique control is conditioned by the requirement:

$$J = \int_0^{t_s} \frac{1}{2}(x_2^2 + u^2)dt \to \min \tag{21}$$

Using the same optimal method, differential equation system as below can be obtained:

$$\dot{x}_1 = x_2; \dot{x}_2 = x3; \dot{x}_3 = u;$$
$$\dot{\lambda}_1 = 0; \dot{\lambda}_2 = x_2 - \lambda_1; \dot{\lambda}_3 = -\lambda_2; \tag{22}$$

By solving the equation system, following equation system is obtained:

$$x_1(t) = C_6 t + C_2\cos t + C_3 e^{t} - C_4 \sin t - C_5 e^{-t} + C_1$$
$$x_2(t) = C_6 + C_2\cos t + C_3 e^{t} + C_4\sin t + C_5 e^{-t}$$
$$x3(t) = -C_2\sin t + C_3 e^{t} + C_4\cos t - C_5 e^{-t} \tag{23}$$
$$u(t) = -C_2\cos(t) + C_3 e^{t} - C_4\sin t + C_5 e^{-t}$$

Where integration constants $C_i (i = 1,...,6)$ are defined by boundary conditions described in (20).

Analysis of Proposed Trajectory

To apply the optimal anti-sway trajectory in practical use, one needs to obtain the relationships related to acceleration or deceleration time, maximum swing angle and maximum acceleration. Analytical result is hard to get from the explicit trajectory functions introduced above. Later in this section, those equation systems can be replaced by further simplified equations. Properties of the proposed trajectory will be

discussed using the further simplified equation.

After the integration constants are determined by boundary conditions, the trajectory of trolley acceleration in jerk period is a sinusoidal shaped function. An example is shown in Fig.2 with parameters $t_j = 3.14$ and $x_1(t_j) = 0.05$.

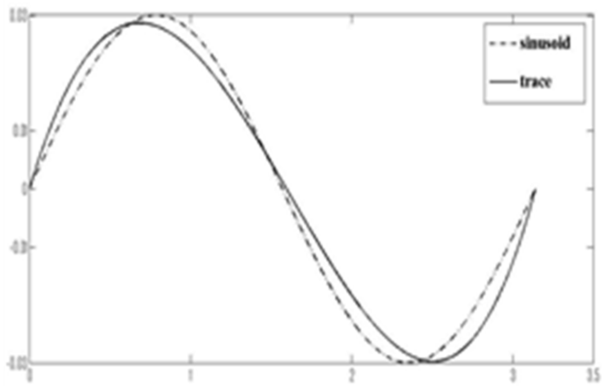

FIG. 2 OPTIMAL TRACE AND SINUSOIDAL FUNCTION

To simplify the result, the trajectory for swing angular acceleration in acceleration phase and that for swing angular velocity in deceleration phase is replaced by a simple sinusoidal function. The simplified sinusoidal trajectory can satisfy all restrictions for anti-sway trajectory although it would induce bigger swing then explicit optimal trajectory. The new equation system for those motion segments and their properties is discussed below respectively.

For jerk period in acceleration phase, the equations are as follow:

$$\ddot{\theta} = -k \sin \omega t \tag{24}$$

$$\theta = \frac{k \sin \omega t}{\omega^2} - \frac{k}{\omega} t \tag{25}$$

By taking equation (24) and (25) into equation (4), one can get the trajectory for $\ddot{x}$:

$$\ddot{x} = \frac{kl \sin \omega t}{\cos(\frac{k \sin \omega t}{\omega^2} - \frac{kt}{\omega})} - g \tan(\frac{k \sin \omega t}{\omega^2} - \frac{kt}{\omega}) \tag{26}$$

The parameter $\omega = 2\pi/t_j$ and $k = 2\pi\theta_a / t_j^2$ in which t_j is the end time of jerk period. The time interval of jerk period should be as short as possible. Given the maximum acceleration of the crane system a_{max} and constant acceleration of uniform acceleration period a_a, the shortest jerk period duration time is

$$t_s^* = \pi\sqrt{2la_a} / \sqrt{ga_{max}} .$$

At the time $t = t_j$, the state of the crane system is as following, in which the swing angle and acceleration will be remind in the uniform acceleration period.

$$\ddot{\theta}(t_j) = 0;\ \dot{\theta}(t_j) = 0;\ \theta(t_j) = -\frac{2k\pi}{\omega^2}; \quad \ddot{x}(t_j) = g \tan(\frac{2k\pi}{\omega^2}) \tag{27}$$

For swing damping period in acceleration phase, the equations are as follow:

$$\ddot{\theta} = k \sin \omega\tilde{t} \tag{28}$$

$$\theta = \theta_a - \frac{k \sin \omega\tilde{t}}{\omega^2} + \frac{k}{\omega}\tilde{t} \tag{29}$$

$$\ddot{x} = -\frac{kl \sin \omega\tilde{t}}{\cos(\theta_a - \frac{k \sin \omega\tilde{t}}{\omega^2} + \frac{k}{\omega}\tilde{t})} - g \tan(\theta_a - \frac{k \sin \omega\tilde{t}}{\omega^2} + \frac{k}{\omega}\tilde{t}) \tag{30}$$

Where $\tilde{t} = 0$ is the beginning of swing damping period. The trajectory of acceleration of trolley in (30) and (26) are obtained with the simplified version of swing trajectory. An optimal version can be obtained by bring equation (13), (15) and (23) into equation (4). Because the equations used in trajectory optimization are intrinsically linear and the equation (4) which is used to derivate acceleration is nonlinear, there is no inaccuracy introduced by linearization. However, the nonlinear form (26) can be linearized into follow form:

$$\ddot{x}(t) = kl \sin \omega t - g\frac{k \sin \omega t}{\omega^2} + g\frac{kt}{\omega} \tag{31}$$

Derived from equation (31), the relation between uniform velocity and acceleration time can be depicted as $v_c = a_a(t_c + t_j)$.

For deceleration phase, the equations are as follow:

$$\theta(t) = -\frac{k \cos wt}{w} + \frac{k}{w} \quad \dot{\theta}(t) = k \sin wt \quad \ddot{\theta}(t) = wk \cos wt \tag{32}$$

where $\omega = 2\pi / t_s$ and $k = 2\pi v_c / gt_s^2$ in which t_s is the time span of deceleration phase, and the largest swing angle is $\theta_{max} = \frac{2k}{\omega} = 2v_c / gt_s$.

The results obtained by optimal method and the simplified version of crane motion are presented in the Fig. 3 and Fig. 4, including: angle of the swing in acceleration phase (Fig. 3a) and in deceleration phase (Fig. 4a); velocity of the point of suspension in acceleration phase (Fig. 3b) and in deceleration phase (Fig. 4b), acceleration of the point of suspension (Fig. 3 c) and in deceleration phase (Fig.4 c). The parameters in this example are: rope length l=6 m, constant velocity $v_c = 2m/s$ jerk time span $t_j = 0.5s$, deceleration time $t_s = 5s$.

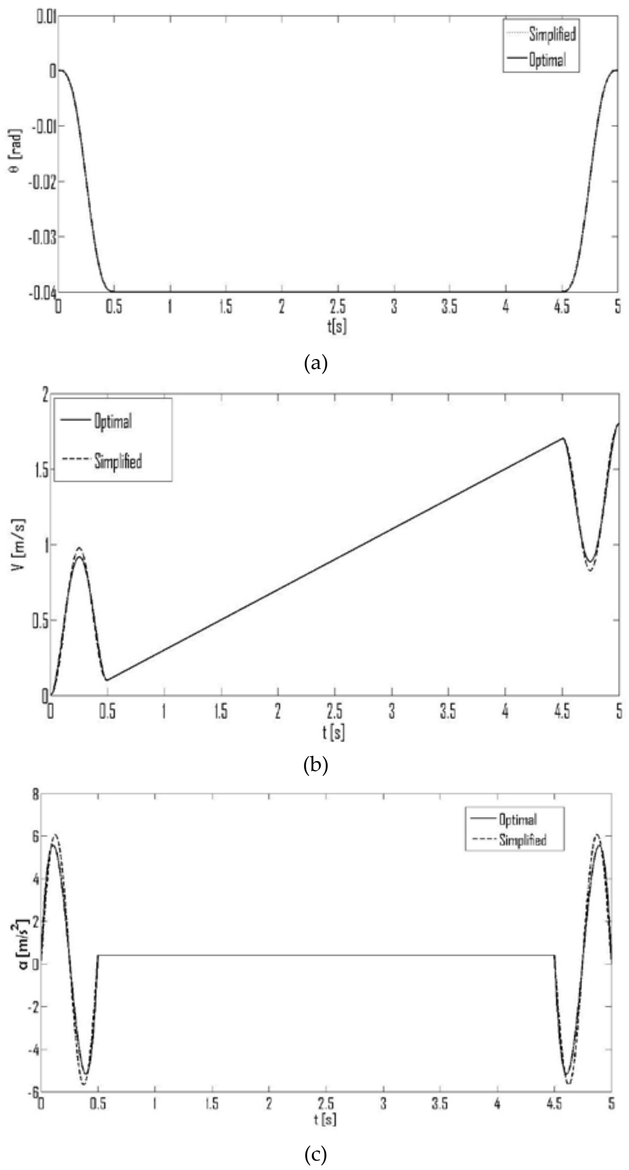

FIG. 3 EXAMPLE OF ACCELERATION PHASE OF OPTIMAL TRAJECTORY

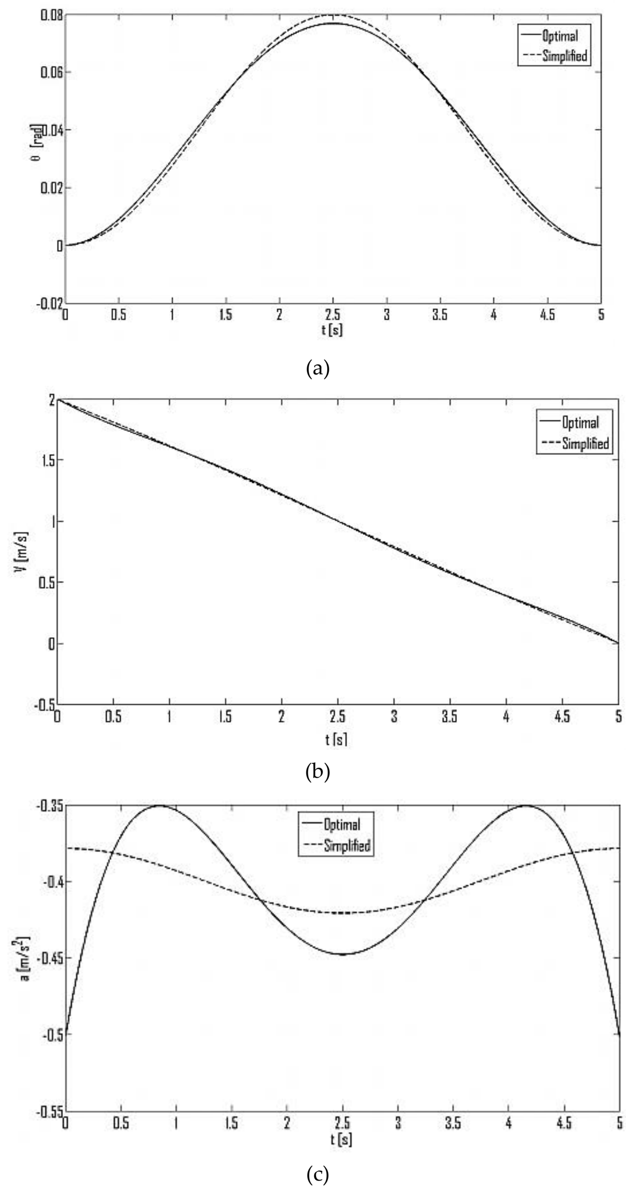

FIG. 4 EXAMPLE OF DECELERATION PHASE OF SIMPLIFIED AND SIMPLIFIED AND OPTIMAL TRAJECTORY

It is worth mentioning that both control functions in section 3 are to prevent swing angle and swing angular acceleration from becoming too high while the actual objective is to minimize the acceleration of the trolley. But from equation (5), it can be observed the difference between acceleration of the trolley and swing angular acceleration is very small. So this replacement is reasonable.

It can be observed in Fig. 3 that acceleration in jerk period is much larger than uniform acceleration value. Usually, this much acceleration cannot be obtained by the trolley. As the deceleration phase is the reverse of acceleration phase, one can use the trajectory of deceleration for acceleration. But it cannot control the trolley into different velocity.

So an independent device with a small inertia should be adopted to produce the extra acceleration.

Conclusions

Two optimal trajectories for crane are proposed which are suitable for acceleration and deceleration respectively. The trajectory would induce minimum swing angle at acceleration and deceleration phase. However, given a known uniform velocity, deceleration trajectory can also be used in acceleration phase. Simplified version of the optimal trajectories is developed which is more convenience for practical use. Also the relationship between accelerated velocity, maximum swing angle, maximum acceleration and accelerate time are discussed using the simplified trajectories. Results obtained in this paper can be applied to anti-sway control for overhead traveling

cranes. An inverse dynamic model for the overhead traveling crane is needed to transform the trajectory into force control input.

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